

Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level FURTHER MATHEMATICS

Paper 1

Exam Date

Morning

Time allowed: 2 hours

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 A vector is given by $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

Which vector is **not** perpendicular to $\mathbf{a}$?

Circle your answer.

[1 mark]

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

2

Use the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x} to show that $\cosh^2 x - \sinh^2 x \equiv 1$

[2 marks]

3 (a) Given that

$$\frac{2}{(r+1)(r+2)(r+3)} \equiv \frac{A}{(r+1)(r+2)} + \frac{B}{(r+2)(r+3)}$$

find the values of the integers A and B

[2 marks]

3 (b) Use the method of differences to show clearly that

$$\sum_{r=9}^{97} \frac{1}{(r+1)(r+2)(r+3)} = \frac{89}{19800}$$

[4 marks]

A student states that $\int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x - \sin x} dx$ is not an improper integral because

$$\frac{\cos x + \sin x}{\cos x - \sin x} \text{ is defined at both } x=0 \text{ and } x=\frac{\pi}{2}$$

Assess the validity of the student's argument.

[illegible]

5 $p(z) = z^4 + 3z^2 + az + b$, $a \in \mathbb{R}$, $b \in \mathbb{R}$

$2 - 3i$ is a root of the equation $p(z) = 0$

5 (a) Express $p(z)$ as a product of quadratic factors with real coefficients.

[5 marks]

5 (b) Solve the equation $p(z) = 0$.

[1 mark]

- 6 (a) Obtain the general solution of the differential equation

$$\tan x \frac{dy}{dx} + y = \sin x \tan x$$

where $0 < x < \frac{\pi}{2}$

[5 marks]

- 6 (b) Hence find the particular solution of this differential equation, given that $y = \frac{1}{2\sqrt{2}}$

when $x = \frac{\pi}{4}$

[2 marks]

- 7 Three planes have equations,

$$x - y + kz = 3$$

$$kx - 3y + 5z = -1$$

$$x - 2y + 3z = -4$$

Where k is a real constant. The planes do not meet at a unique point.

- 7 (a) Find the possible values of k

[3 marks]

Identify each possible configurations, stating the corresponding value of k

[5 marks]

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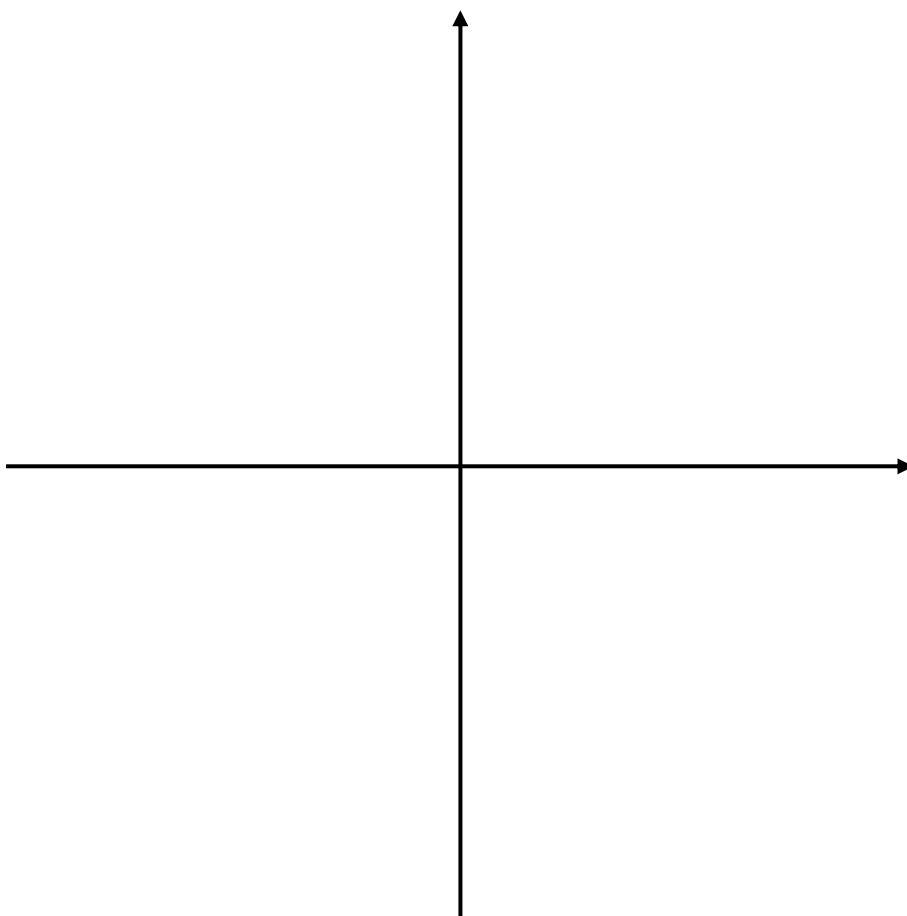
[3 marks]

8 A curve has equation

$$y = \frac{5 - 4x}{1 + x}$$

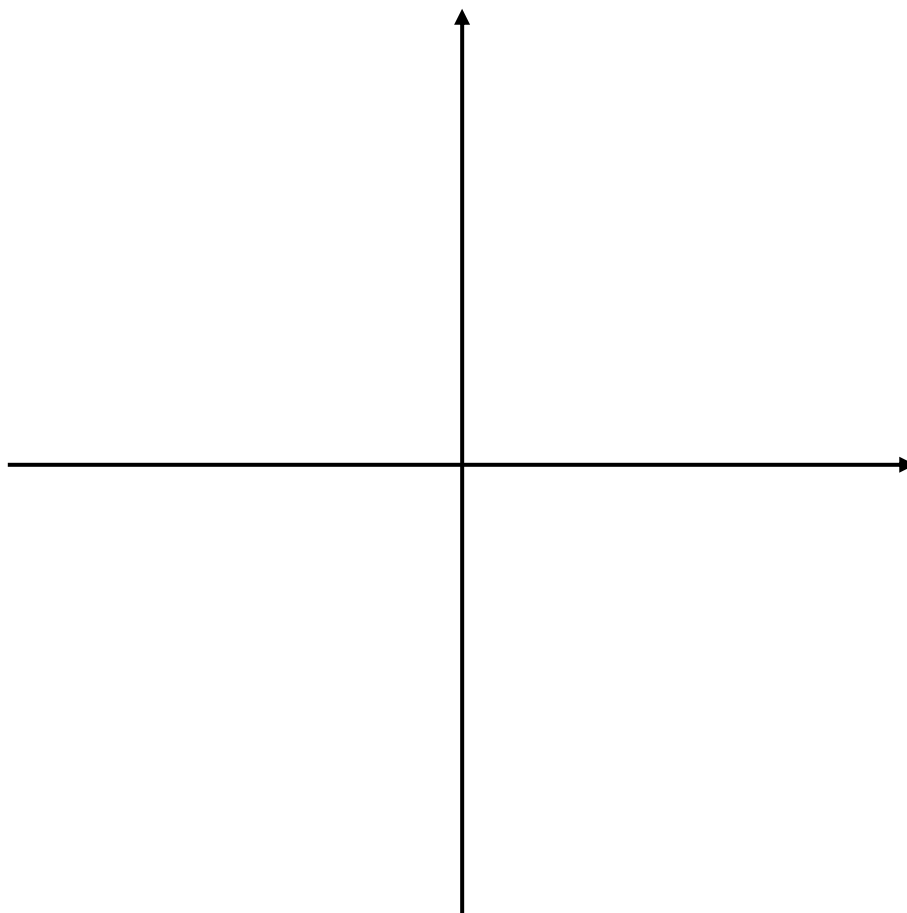
8 (a) Sketch the curve.

[4 marks]



8 (b) Hence sketch the graph of $y = \left| \frac{5 - 4x}{1 + x} \right|$.

[1 mark]



- 9 A line has Cartesian equations $x - p = \frac{y + 2}{q} = 3 - z$ and a plane has

equation $\mathbf{r} \cdot \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = -3$

- 9 (a) In the case where the plane fully contains the line, find the values of p and q .

[3 marks]

- 9 (b) In the case where the line intersects the plane at a single point, find the range of values of p and q .

[3 marks]

-
- 9 (c)** In the case where the angle θ between the line and the plane satisfies $\sin\theta = \frac{1}{\sqrt{6}}$ and the line intersects the plane at $z = 0$

- 9 (c) (i)** Find the value of q .

[4 marks]

- 9 (c) (ii)** Find the value of p .

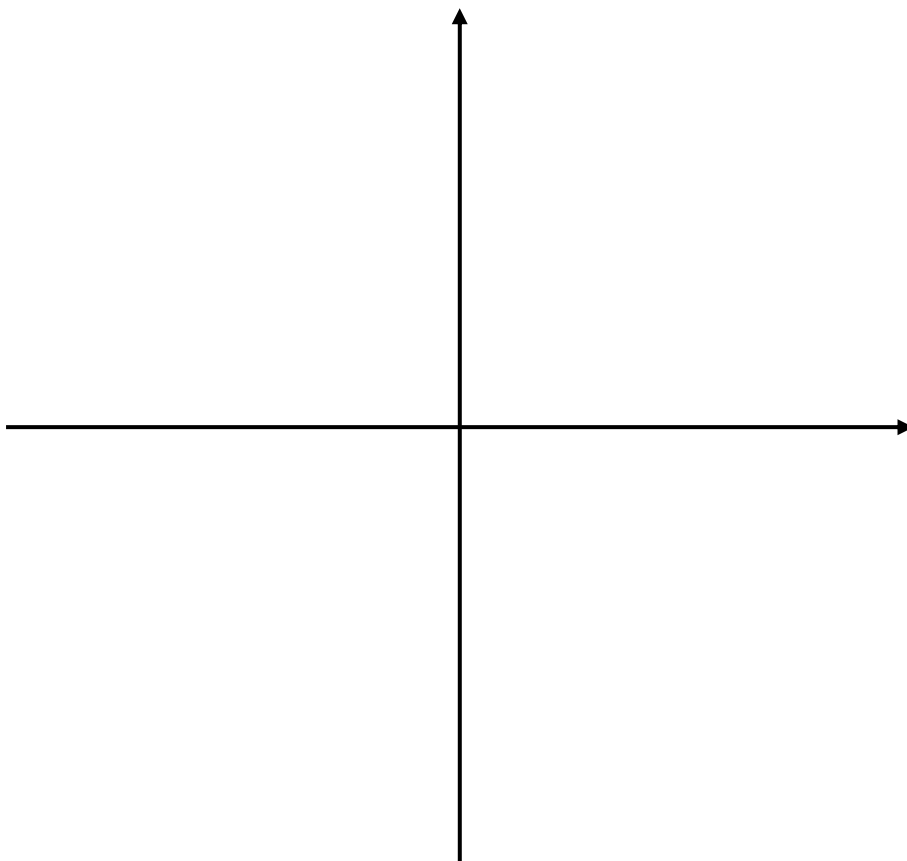
[3 marks]

10 The curve, C , has equation $y = \frac{x}{\cosh x}$

10 (a) Show that the x -coordinates of any stationary points of C satisfy the equation $\tanh x = \frac{1}{x}$
[3 marks]

10 (b) (i) Sketch the graphs of $y = \tanh x$ and $y = \frac{1}{x}$ on the axes below.

[2 marks]



[1 mark]

[4 marks]

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11 (a) Prove that $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} \equiv 2 \coth \theta$

Explicitly state any hyperbolic identities that you use within your proof.

[4 marks]

11 (b) Solve $\frac{\sinh \theta}{1 + \cosh \theta} + \frac{1 + \cosh \theta}{\sinh \theta} = 4$ giving your answer in an exact form.

[2 marks]

The function $f(x) = \cosh(ix)$ is defined over the domain $\{x \in \mathbb{R} : -a\pi \leq x \leq a\pi\}$, where a is a positive integer.

By considering the graph of $y = [f(x)]^n$, find the mean value of $[f(x)]^n$, when n is an odd positive integer.

Fully justify your answer.

[illegible]

Given that $\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $\mathbf{M}^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ for all $n \in \mathbb{N}$

[illegible]

- Fully justify your answer.

[illegible]

14 (b) It is decided to alter the resistive force so that the motion of P is critically damped.

Determine the magnitude of the resistive force that will produce critically damped motion.

[4 marks

It is assumed that:

- 15 (a) (i)** Construct a mathematical model for the number of rabbits.

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[1 mark]

- 15 (b)** Suggest one way in which the model that you have used for the number of rabbits could be refined. **[1 mark]**

END OF QUESTIONS